Even powers of sine and/or cosine

First, we need to build a couple of trigonometric tools

Recall the double-angle formula: $\cos 2x = \cos^2 x - \sin^2 x$

Substitute $\sin^2 x = 1 - \cos^2 x$ into the formula above and re-arrange to get:

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

Substitute $\cos^2 x = 1 - \sin^2 x$ in instead and get:

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

Example 1

$$\int \sin^2 x \, dx = \int \frac{1}{2}(1 - \cos 2x)dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

Example 2

$$\int \cos^2(3x) \, dx = \int \frac{1}{2}(1 + \cos 6x)dx = \frac{1}{2}x + \frac{1}{12}\sin 6x + C$$

You try (answers on the next page)

1. $\int \cos^2(5x) \, dx$

2. $\int \sin^2(4x) \, dx$

3. $\int \cos^2\left(\frac{1}{2}x\right) dx$

Answers

1. $\frac{1}{2}x + \frac{1}{20}\sin 10x + C$

2. $\frac{1}{2}x - \frac{1}{16}\sin 8x + C$

3. $\frac{1}{2}x + \frac{1}{2}\sin x + C$